Induced Drag at a Glance

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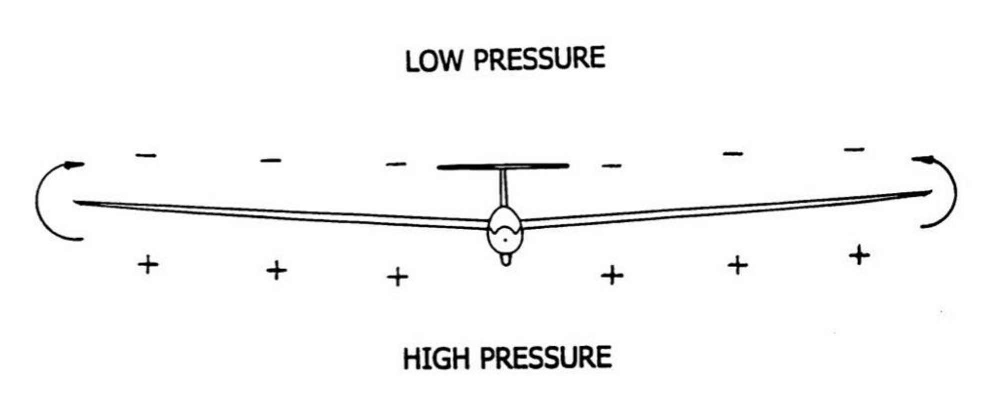
0. Summary

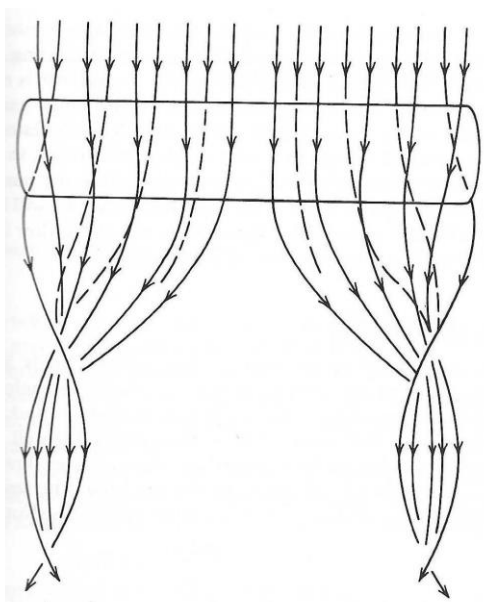
- Lift arises from the difference in pressure between the top of the airfoil (where the pressure is less than ambient) and the bottom (where the pressure is higher than ambient). The pressure difference creates a flow around the wingtip (from lower to upper surface), which eventually results in two vortices ("wingtip vortices") trailing the wing. These vortices induce a downward velocity ahead of and behind the wing—the downwash. At the wing location, this downwash reduces the angle of attack (reducing the lift), and tilts the lift backward, introducing a drag component—the *induced drag*. The power dissipated by the induced drag is converted into the rotational motion of the wingtip vortices, and is responsible for the intensity of "wake turbulence."
- In the early years of the 20th century, Ludwig Prandtl derived a simplified model for the wake vortices and resultant induced velocities—the so-called Lifting Line Theory. Some important results from the Lifting Line theory are:
  - The induced drag coefficient is proportional to the square of the lift coefficient (i.e., induced drag increases rapidly with lift), and inversely proportional to the wing aspect ratio.
  - Induced drag increases rapidly with aircraft weight
  - Induced drag increases rapidly with decreasing airspeed
  - The energy content of the wake vortices being a direct result of induced drag, wake turbulence intensity will increase rapidly with increased weight and with decreased airspeed. This outlines the fact that wake turbulence is at its most dangerous for heavy aircrafts flying at low speeds.
  - At low speeds, induced drag is greater than parasite drag. The reverse is true at high speeds.
  - Specifically, at higher speeds (roughly above *Vy*), the parasite drag contribution dominates and increases as the third power of speed—going fast is expensive (a 10% increase in speed will roughly require a 30% increase in power—hence fuel flow!).
  - At lower speeds (roughly below *Vy*), the parasite drag contribution decreases rapidly, but the induced drag component keeps increasing. That's why below a certain airspeed (roughly the best climb angle speed), the power required to maintain straight and level flight increases with decreasing airspeed—we fly "behind the power curve."
- When flying close to the ground, the presence of the ground surface interferes with the downwash development, reducing its effect on lift and drag. This results in increased performance—the "ground effect."
- Longitudinal stability of an airplane requires the lift of the stabilizer to be acting in the downward direction. The wing will thus have to develop extra lift—the weight of the aircraft plus the stabilizer lift. This increased lift results in an increase in induced drag—this increase is called the "trim drag."
- The main function of winglets is to reduce induced drag. This however comes at the cost of increased parasite drag due to the extra surface area.

1. The Physical Picture

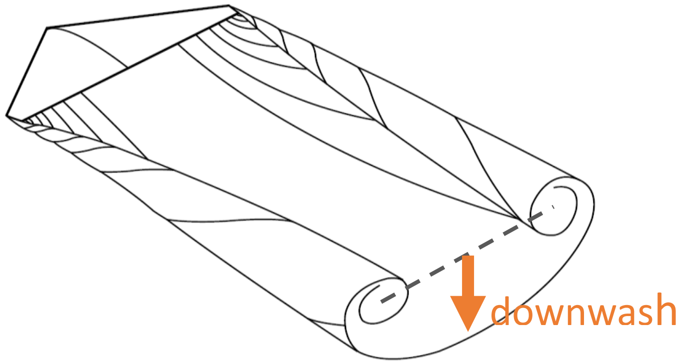
Lift arises from the difference in pressure between the top of the airfoil (where the pressure is less than ambient) and the bottom (where the pressure is higher than ambient).



This pressure difference causes a flow around the wingtip, as shown above.

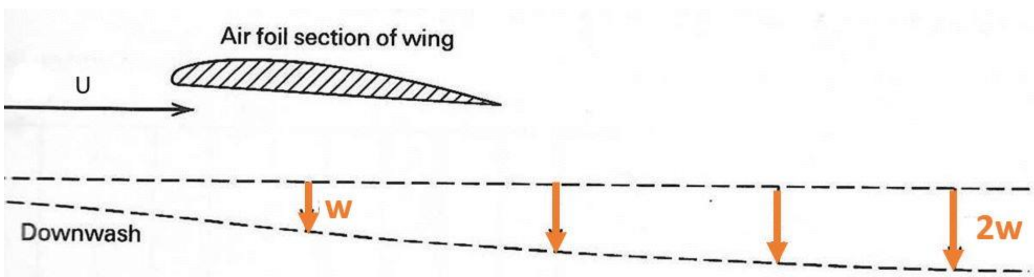


The flow around the wingtip causes a spanwise flow towards the wingtip at the lower surface (dashed streamlines above), and towards the center of the wing at the upper surface (solid lines). Combined with the freestream velocity (longitudinal direction), this spanwise flow creates a swirling motion downstream of the wing, resulting in two trailing vortices. Let's note that, while these vortices are commonly referred to as "wingtip vortices," they origin spans the entire wingspan.

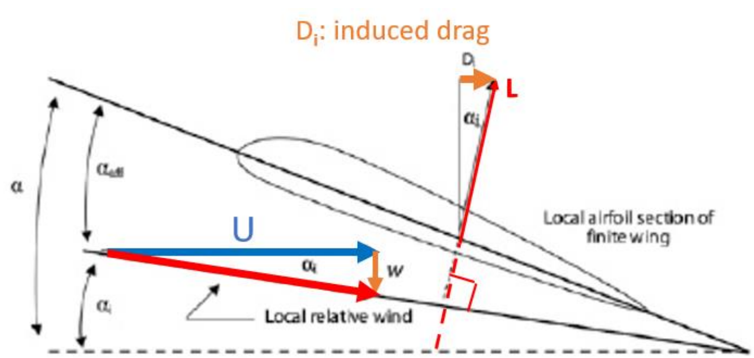


The above figure shows a 3D representation of the trailing vortices. The left vortex rotates in a clockwise direction (looking in the direction of flight), and the right vortex rotates in a counterclockwise direction. That motion induces a downward velocity in-between the vortices—the downwash (in orange).

Looking at the wing from the side, far from ahead of the wing, the downwash vanishes. Far downstream, the downwash approaches a value twice that induced at the wing (*w*).



The downwash at the wing, *w*, will combine with the freestream velocity *U*, resulting in the relative velocity experienced by the wing section, as illustrated below, where the resultant relative velocity is shown in red. The corresponding lift will be perpendicular to this relative velocity (vector labeled *L*). This vector has a component *Di* lying along freestream velocity *U*. This force is opposing the motion of the aircraft—it is a drag component associated with the induced velocity, the so-called *induced drag*. Also, the angle of attack of the wing with respect to the local relative velocity is smaller than the one relative to the freestream velocity. The lift will thus be reduced compared to the ideal two-dimensional case.



The power dissipated by the induced drag (= *U* * *Di*) is converted into the rotational motion of the wake vortices, and is responsible for the intensity of "wake turbulence."

2. Prandtl's Model and its Consequences

In the early years of the 20th century, Ludwig Prandtl derived a simplified model for the wake vortices and resultant induced velocities—the so-called Lifting Line Theory. The main result of that theory is a simple expression for the induced drag coefficient C_{Di}:

$$C_{Di} = \frac{C_L^2}{\pi A e}$$

Where $C_L$is the lift coefficient, A the wing aspect ratio, and *e* an efficiency factor based on the geometry of the wing. It is equal to 1 for a wing of elliptical planform, and is less than one for other planforms (i.e., the elliptical wing produces the least possible induced drag compared to other planform).

If we combine this expression for the induced drag coefficient with the usual expressions for lift (*L*) and drag (*D*), with *S* the surface area of the wing and *W* the weight of the aircraft:

$$L = W = \frac{1}{2}\rho V_L^2 S C_L \text{ and } D_i = \frac{1}{2}\rho V^2 S C_{Di}$$

We obtain the following result:

$$D_i = K\frac{W^2}{V^2}$$

Where K incorporates the geometry of the aircraft. This brings us to the following conclusions:

- **Induced drag increases rapidly with aircraft weight**
- **Induced drag increases rapidly with decreasing airspeed**
- **The energy content of the wake vortices being a direct result of induced drag, wake turbulence intensity will increase rapidly with increased weight and with decreased airspeed. This outlines the fact that wake turbulence is at its most dangerous for heavy aircrafts flying at low speeds.**

How does induced drag compare to parasite drag? A useful rule of thumb is that, at *Vy* (best climb rate airspeed), the induced drag is approximatively equal to the parasite drag. Since induced drag increases with decreasing airspeed, we can thus conclude that

- **At low speeds, induced drag is dominant**
- **At high speeds, parasite drag is dominant**

If we now calculate the total power needed to balance total drag, assuming the parasite drag coefficient to be constant, we obtain the following expression (K_0 and K_0 are constants):

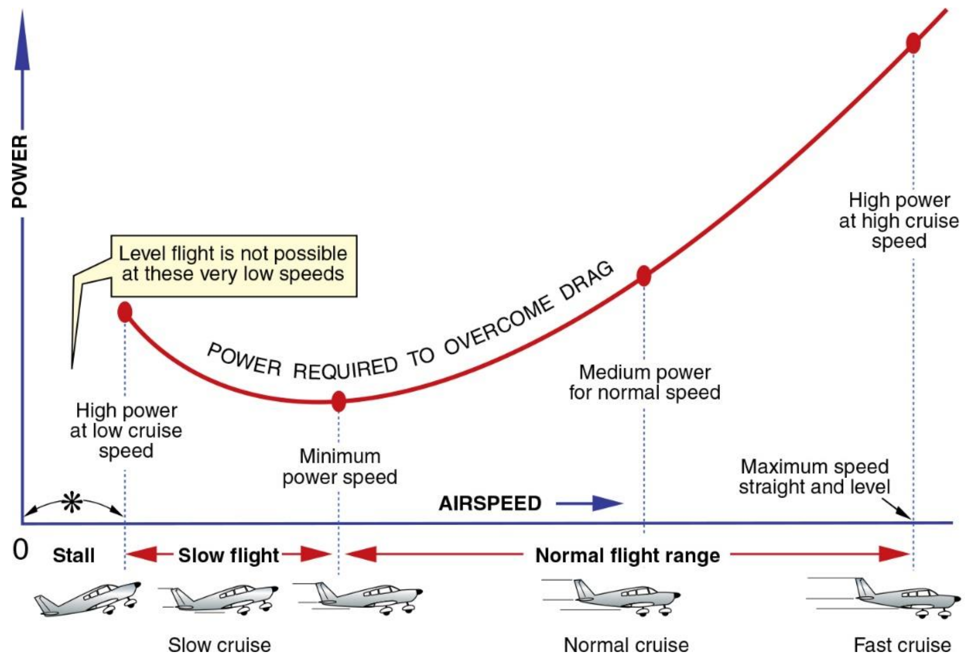
$$Power = K_0 V^3 + \frac{K_i}{V}$$

So, the power required for straight-and-level flight consist of a term proportional to the cube of airspeed (the parasite drag contribution), and a term proportional to the inverse of airspeed (the induced drag contribution).

- **At higher speeds, the parasite drag contribution dominates, increasing as the third power of speed—going fast is expensive (a 10% increase in speed will roughly require a 30% increase in power—hence fuel flow!).**

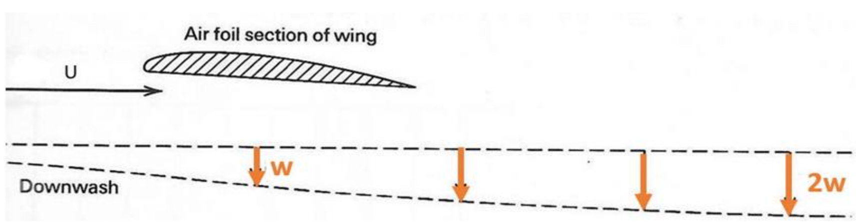
- **At lower speeds though, the parasite drag contribution decreases rapidly, while the induced drag component increases. That's why below a certain airspeed (roughly the best climb angle speed), the power required to maintain straight and level flight increases—we fly "behind the power curve."**

The power required for straight-and-level flight is illustrated below:

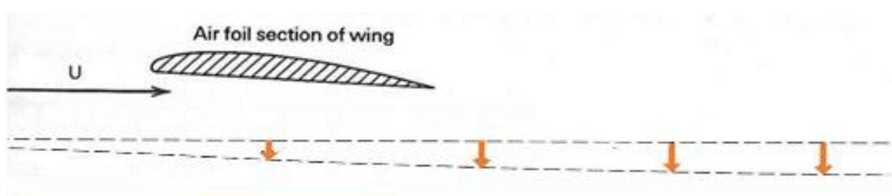


3. Ground effect

As we have seen in the first section, induced drag, and loss of lift, are associated with the downwash induced by the wake vortices. The downwash velocities are represented below.

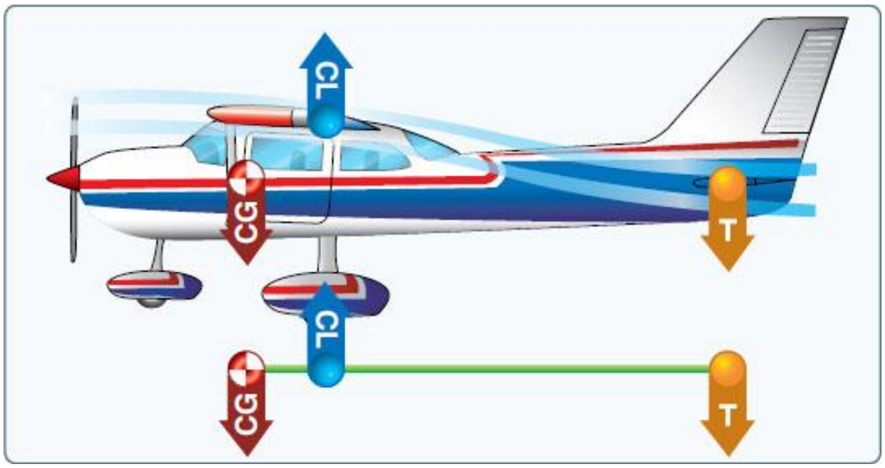


If the aircraft flies close to the ground though, the ground surface will interfere with the development of the downwash, the downwash velocities will be reduced, and so will the induced drag and the loss of lift.



This is why, when flying close to the ground, we experience increased performance and a tendency to "float"—the ground effect. This increase in performance is not actually a good thing in our everyday flying—when landing, it increases the landing distance, and when taking off, it might tempt us into trying to climb too early…

4. Trim Drag



A wing by itself is not stable—any increase in the angle of attack (due to a gust for example) will generate a moment that will tend to further increase it.

The traditional way to stabilize the system is to move the CG ahead of the center of lift of the wing, and add a stabilizer surface downstream of the wing, with a downward lift. As a result, the wing will have to provide additional lift (weight + T, vs. weight only). Consequently, the induced drag will be increased also. The more forward the CG is, the higher the induced drag penalty.

There are two ways to alleviate this issue. One is to switch to a canard configuration (e.g., Burt Rutan's LongEZ)—in this case the stabilizer will carry an upward lift. But stall characteristics and stability limitations limit the widespread adoption of this configuration. Another approach is to carry upward lift on a conventional stabilizer, and sacrifice longitudinal stability—an approach only doable with fly-by-wire technology and high-speed stability-augmenting control systems (e.g., F16). This is not (yet…) an option for GA airplanes.

5. Winglets

Winglets seems to sprout on wingtips everywhere, from airliners to gliders. Why?

Winglets allow the reduction of induced drag. Only a detailed flow analysis can provide exact gains, but a qualitative analysis indicates that they allow the recovery of some of the energy that would be otherwise be dissipated in the wingtip vortices.

This gain in induced drag comes however with an associated cost—the increased parasite drag due to the additional surface area.

Winglets will:

- Improves low speed drag, where induced drag is dominant
- Increases drag at high speeds, where parasite drag is dominant

Fact: a wingspan extension equal to the height of the winglet will produce a greater induced drag reduction than the winglets! So, why using winglets? There are two main reasons:

- Structural considerations
    - winglets have a smaller impact on bending moments and shear stresses than wing extensions
- Wing span considerations
    - ICAO classifies airplanes in discrete classes based on wingspan. Moving from one class to a higher one increases ramp costs for the airline. Winglets allow decreasing induced drag without increasing wingspan.
    - Gliders are divided in classes based on wingspan—15m class, 18m class, 20 m class. Winglets provide the potential of reduced drag (at least at low speed) within the wingspan constraint.